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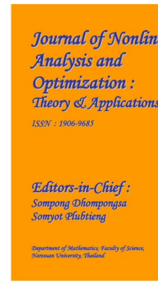


# A COMPARATIVE STUDY BETWEEN INDEX OF DISPERSION AND MDMA METHOD- AN OPTIMAL SOLUTION FOR TRANSPORTATION PROBLEM

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## ABSTRACT

The transportation problem is a classic optimization challenge in operations research and logistics, aiming to minimize the cost of distributing goods from multiple suppliers to multiple destinations. In this article, we are comparing two methods which are used to get initial basic feasible solution of transportation problem. The two methods were MDMA (Maximum Divide Minimum Allotment) method and Index of Dispersion method. Finally, comparing solution of these methods with NWCR, LCM and VAM.
**Keywords:** Transportation Problem, MDMA, Index of Dispersion, NWCR, VAM, LCM.

## Introduction

The transportation problem is a fundamental issue in operations research and logistics, playing a crucial role in the efficient allocation of resources in various industries. It involves determining the optimal way to transport goods from multiple suppliers to multiple destinations, taking into account factors such as transportation costs, capacities and demands. This problem arises in a wide range of real-world scenarios, including distribution networks, supply chain management, and urban transportation systems.

In order to address the transportation problem, it can be mathematically expressed as a linear programming model. Various solution techniques, including the North – West Corner Method, Least Cost Method, Vogel's Approximation Method, and Modified Distribution Method, employ iterative allocation rules to optimize the assignment of goods and achieve an optimal solution.

The index of Dispersion refers to s statistical measure that quantifies the variability in a dataset relative to its average. It is calculated by dividing the variance of the dataset by its mean. The variance measures the dispersion or spread of the data points, while the mean represents the average value. By computing the Index of Dispersion, we can gain insights into the degree of variability present in the dataset relative to its central tendency. In the context of the transportation problem, the Index of Dispersion can be utilized as a criterion to assess and compare different solutions or approaches, potentially aiding in the selection of the most suitable method for achieving an initial basic feasible solution.

**Transportation Problem Through MDMA (Maximum Divide Minimum Allotment) Method:**

A. Amaravathy, K. Thiagarajan and S. Vimala [2]
The MDMA method proceeds as follows:
Step 1: Construct the Transportation Table for the given pay off matrix.
Step 2: Choose the maximum element from pay off matrix and divide all elements by the maximum element in the constructed transportation table.
Step 3: Supply the demand for the minimum element newly constructed transportation table.
Step 4: Select the next maximum element in constructed transportation table and repeat the same procedure for remaining allotments.

**Algorithm for Index of Dispersion** Ajay Paul and Vishal vincet henry [1]
To determine the basic feasible solution for the transportation problem is presented as follows:
Step 1: Verify if the transportation problem is balanced by checking if the total supply is equal to the total demand. If the transportation is not balanced, balance it by adding a dummy row or column to adjust the supply and demand values.
Step 2: Determine the variance $\sigma^2 = \sum_{i=1}^{n} (x_i - \bar{x})^2 / n-1$ and the mean $\bar{x_i} = \sum_{i=1}^{n} x_i / n$ for each row. Then obtain the respective Index of Dispersion is, $D = \frac{\sigma^2}{\bar{x_i}}$ and perform the multiplication of the obtained value with its corresponding supply value ( D * Supply = $D_s$).
Step 3: Determine the variance $\sigma^2 = \sum_{i=1}^{n} (x_i - \bar{x})^2 / n-1$ and the mean $\bar{x_i} = \sum_{i=1}^{n} x_i / n$ for each column. Then obtain the respective Index of Dispersion is, $D = \frac{\sigma^2}{\bar{x_i}}$ and perform the multiplication of the obtained value with its corresponding demand value (D * Demand = $D_D$).
Step 4: Determine the row or column that exhibits the highest $D_S$ or $D_D$ among all the rows and columns, resolving ties arbitrarily. Find the cell within the chosen row or column that has the lowest cost and allocate as many units as a feasible to that cell.
Step 5: Subtract the number of units assigned to the cell from the row supply and column demand, and the mark the row supply or column demand as satisfied. Created a new table based on this adjustment. If both a row and a column are satisfied at the same time, mark only one of them as satisfied and assign a zero demand (or supply) to the remaining column ( or row). Additionally, when calculating subsequent Index of Dispersion, do not include any column or row with zero demand or supply.
Step 6: Recomputed[ $D_S$ or $D_D$] for each row and columns in the reduced transportation table, as outlined in steps 2 and 3. Proceed to steps 4 and 5 accordingly. Repeat this iterative process until all the demands and supplies are fulfilled.
Step 7: Lastly, compute the total transportation cost of the transportation table. This calculation involves summing the product of the cost and the corresponding assigned value for each cell in the transportation table.

**Numerical Example**

| Source | Destination | | | | Supply |
|---|---|---|---|---|---|
| | A | B | C | D | |
| X | 3 | 1 | 7 | 4 | 250 |
| Y | 2 | 6 | 5 | 9 | 350 |
| Z | 8 | 3 | 3 | 2 | 400 |
| Demand | 200 | 300 | 350 | 150 | |

**Solution:**

**Method 1: MDMA method**

Consider the transportation problem:

| Source | Destination | | | | Supply |
|---|---|---|---|---|---|
| | A | B | C | D | |
| X | 3 | 1 | 7 | 4 | 250 |
| Y | 2 | 6 | 5 | 9 | 350 |
| Z | 8 | 3 | 3 | 2 | 400 |
| Demand | 200 | 300 | 350 | 150 | |

Step 1: Here the maximum element is 9, Divide all elements by ME=9

| Source | Destination | | | | Supply |
|---|---|---|---|---|---|
| | A | B | C | D | |
| X | 3/9 | 1/9 [250] | 7/9 | 4/9 | ~~250~~ |
| Y | 2/9 | 6/9 | 5/9 | 9/9 | 350 |
| Z | 8/9 | 3/9 | 3/9 | 2/9 | 400 |
| Demand | 200 | ~~300~~[50] | 350 | 150 | |

Select the minimum element (1/9), allot the minimum demand and allot [D(300),S(250)] =250 unit in the cell (1,2).

Step 2: Choose the next maximum element is 8/9, Divide all elements by ME = 8/9

| Source | Destination | | | | Supply |
|---|---|---|---|---|---|
| | A | B | C | D | |
| Y | 2/8 [200] | 6/8 | 5/8 | 9/8 | ~~350~~[150] |
| Z | 1 | 3/8 | 3/8 | 2/8 | 400 |
| Demand | ~~200~~ | 50 | 350 | 150 | |

Select the minimum element (2/8), allot the minimum demand and allot [D(200),S(350)] =200 unit in the cell (1,1).

Step 3: Choose the next maximum element is 9/8, Divide all elements by ME = 9/8

| Source | Destination | | | Supply |
|---|---|---|---|---|
| | B | C | D | |
| Y | 6/9 | 5/9 | 1 | 150 |
| Z | 3/9 | 3/9 | 2/9[150] | ~~400~~ [250] |
| Demand | 50 | 350 | ~~150~~ | |

Select the minimum element (2/9), allot the minimum demand and allot [D(150),S(400)] =150 unit in the cell (2,3).

Step 4: Choose the next maximum element is 6/9, Divide all elements by ME = 6/9

| Source | Destination | | Supply |
|---|---|---|---|
| | B | C | |
| Y | 1 | 5/6 | 150 |
| Z | 3/6 | 3/6 [250] | ~~250~~ |
| Demand | 50 | ~~350~~ [100] | |

Select the minimum element (3/6), allot the minimum demand and allot [D(350),S(250)] =250 unit in the cell (2,2).

Step 5: Choose the next maximum element is 1, Divide all elements by ME = 1

| Source | Destination | | Supply |
|---|---|---|---|
| | B | C | |
| Y | 1 | 5/6 [100] | ~~150~~ [50] |
| Demand | 50 | ~~100~~ | |

Select the minimum element (5/6), allot the minimum demand and allot [D(100),S(150)] =100 unit in the cell (1,2).

Step 6:

| Source | Destination | Supply |
|---|---|---|
| | B | |
| Y | 1 [50] | ~~50~~ |
| Demand | ~~50~~ | |

Finally choose the minimum demand and allot 50 unit in cell (1,1) which leads to the solution Satisfying all the constraints. The resulting initial basic feasible solution is given below

Step 7:

| Source | Destination | | | | Supply |
|---|---|---|---|---|---|
| | A | B | C | D | |
| X | 3 | 1 [250] | 7 | 4 | 250 |
| Y | 2 [200] | 6 [50] | 5 [100] | 9 | 350 |
| Z | 8 | 3 | 3 [250] | 2 [150] | 400 |
| Demand | 200 | 300 | 350 | 150 | |

The cost is = 1*250 + 2*200 + 6*50 + 5*100 + 3*250 + 2*150
= 250 + 400 + 300 + 500 + 750 + 300
= 2500

**Method 2: Index of Dispersion**
Consider the transportation problem

| Source | Destination | | | | Supply |
|---|---|---|---|---|---|
| | A | B | C | D | |
| X | 3 | 1 | 7 | 4 | 250 |
| Y | 2 | 6 | 5 | 9 | 350 |
| Z | 8 | 3 | 3 | 2 | 400 |
| Demand | 200 | 300 | 350 | 150 | |

**Solution:**
Step 1:

| Source | Destination | | | | Supply | D | $D_S$ |
|---|---|---|---|---|---|---|---|
| | A | B | C | D | | | |
| X | 3 | 1 | 7 | 4 | 250 | 2.08 | 520.8 |
| Y | 2 | 6 | 5 | 9 | 350 | 1.5 | 530.3 |
| Z | 8 | 3 | 3 | 2 [150] | ~~400~~ [250] | 1.8 | 733.3 |
| Demand | 200 | 300 | 350 | ~~150~~ | | | |
| D | 2.4 | 1.9 | 0.8 | 2.6 | | | |
| $D_D$ | 480 | 576 | 280 | 390 | | | |

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Step 2:

| Source | Destination | | | Supply | D | $D_S$ |
|---|---|---|---|---|---|---|
| | A | B | C | | | |
| X | 3 | 1 | 7 | 250 | 2.5 | 625 |
| Y | 2 [200] | 6 | 5 | ~~350~~ [150] | 4.3 | 1505 |
| Z | 8 | 3 | 3 | 250 | 1.8 | 450 |
| Demand | ~~200~~ | 300 | 350 | | | |
| D | 2.4 | 1.9 | 0.8 | | | |
| $D_D$ | 480 | 576 | 280 | | | |

Step 3:

| Source | Destination | | Supply | D | $D_S$ |
|---|---|---|---|---|---|
| | B | C | | | |
| X | 1 [250] | 7 | ~~250~~ | 4.5 | 1125 |
| Y | 6 | 5 | 150 | 0.1 | 15 |
| Z | 3 | 3 | 250 | 0 | 0 |
| Demand | ~~300~~ [50] | 350 | | | |
| D | 1.9 | 0.8 | | | |
| $D_D$ | 576 | 280 | | | |

Step 4:

| Source | Destination | | Supply | D | $D_S$ |
|---|---|---|---|---|---|
| | B | C | | | |
| Y | 6 | 5 | 150 | 0.1 | 15 |
| Z | 3 | 3 [250] | ~~250~~ | 0 | 0 |
| Demand | 50 | ~~350~~ [100] | | | |
| D | 1 | 0.5 | | | |
| $D_D$ | 50 | 175 | | | |

Step 5:

| Source | Destination | | Supply | D | $D_S$ |
|---|---|---|---|---|---|
| | B | C | | | |
| Y | 6 | 5 [100] | 150 [50] | 0.1 | 15 |
| Demand | 50 | ~~100~~ | | | |
| D | 0 | 0 | | | |
| $D_D$ | 0 | 0 | | | |

Step 6:

| Source | Destination | Supply |
|---|---|---|
| | B | |
| Y | 6 [50] | ~~50~~ |
| Demand | ~~50~~ | |

In the end, we allocate 50 units to the leftover cost in transportation problem.
Total cost = 6*50 + 5*100 + 3*250 + 1*250 + 2*200 + 2*150
= 300 + 500 + 750 + 250 + 400 + 300
= 2500
By observing that the total number of allocations is 6 in the transportation problem equals to m+n-1, i.e., 3+4-1 = 6 we can infer that the solution is non-degenerate.

**Comparison of result**

| **Method** | **Solution** |
|---|---|
| LCM | 2500 |
| NWCR | 3700 |
| VAM | 2450 |
| MDMA | 2500 |
| Index of Dispersion | 2500 |

**Conclusion**

In this article, we have compared two methods mainly MDMA method and Index of dispersion method with NWCR, LCM and VAM which are all used to get initial basic feasible solution. Most probably, all these methods give nearly optimal solution, it is easy to understand and remember better. In summary, these two approaches offer an improved Initial Basic Feasible Solution, ensuring minimal costs in most of the problems.

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